Law of truly large numbers

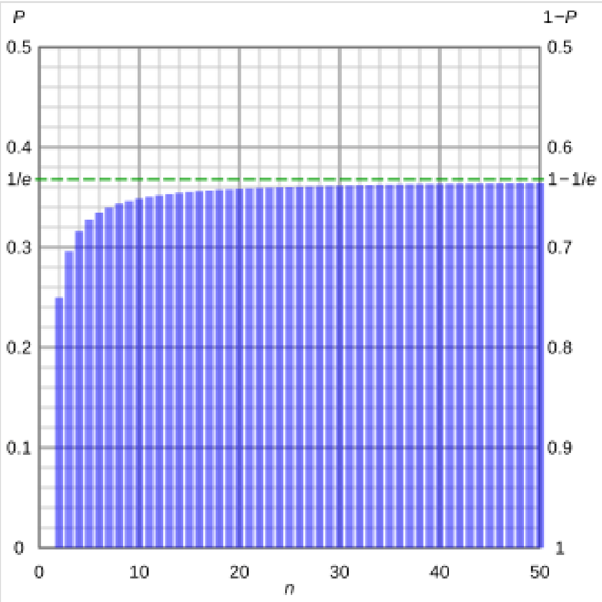
The **law of truly large numbers** is the observation in statistics that any highly unlikely result (i.e., an event with constantly low but non-zero probability across samples) is likely to occur, given a large enough number of independent samples. It is not a mathematical law, but a colloquialism.[1] The law has been used to rebut pseudo-scientific claims.[2]

The observation is attributed to statisticians Persi Diaconis and Frederick Mosteller.[1] Skeptic and magician Penn Jillette similarly said that "million-to-one odds happen eight times a day" among the roughly 8 million inhabitants of New York City.[3] In another illustrative class of cases—which also involve combinatorics—lottery drawing numbers have been duplicated in close or even immediate succession.[4][5][6]

Examples

Suppose that an event A has only a 1% probability of occurring in a single trial. Then, within a single trial, there is a 99% probability that A will not occur. However, if 100 independent trials are performed, the probability that A does not occur in a single of them, even once, is $0.99^{100} \approx 36.6\%$.[7] Therefore, probability of A occurring in at least one of 100 trials is $1 - 0.99^{100} \approx 63.4\%$. If the number of trials is increased to 1,000, that probability rises to $1 - 0.99^{1000} \approx 99.997\%$. In other words, a highly unlikely event, given enough independent trials, is very likely to occur.

Similarly, for an event B with "one in a billion odds" of occurring in any single trial, across 1 billion independent trials the probability of B occurring at least once is $1 - 0.999999999^{1{,}000{,}000{,}000} \approx 63.21\%$. Taking a "truly large" number of independent trials like 8 billion (the approximate human population of Earth as of 2022) raises this to 99.96%.[8]



Graphs of probability *P* of *not* observing independent events each of probability 1/*n* after *n* Bernoulli trials, and 1 − *P* vs *n*. As *n* increases, the probability of a 1/*n*-chance event never appearing after *n* tries rapidly converges to 1/*e*.

These calculations can be formalized in mathematical language as: "the probability of an unlikely event X happening in N independent trials can become arbitrarily near to 1, no matter how small the probability of the event X in one single trial is, provided that N is truly large."[9]

For example, where the probability of unlikely event X is not a small constant but decreased in function of N, see graph.

In high availability systems even very unlikely events have to be taken into consideration, in **series** systems even when the probability of failure for single element is very low after connecting them in large numbers probability of whole system failure raises (to make system failures less probable redundancy can be used — in such **parallel** systems even highly unreliable redundant parts connected in large numbers raise the probability of not breaking to required high level).[10]

In criticism of pseudoscience

The law comes up in criticism of pseudoscience and is sometimes called the Jeane Dixon effect (see also Postdiction). It holds that the more predictions a psychic makes, the better the odds that one of them will "hit". Thus, if one comes true, the psychic expects us to forget the vast majority that did not happen which is called the confirmation bias.[11] Humans can be susceptible to this fallacy.

Another similar manifestation of the law can be found in gambling, where gamblers tend to remember their wins and forget their losses,[12] even if the latter far outnumber the former (though depending on a particular person, the opposite may also be true when they think they need more analysis of their losses to achieve fine tuning of their playing system[13]). Mikal Aasved links it with "selective memory bias", allowing gamblers to mentally distance themselves from the consequences of their gambling[13] by holding an inflated view of their real winnings (or losses in the opposite case – "selective memory bias in either direction").

See also

- Law of large numbers – Averages of repeated trials converge to the expected value
- Poisson clumping
- Black swan theory – Theory of response to surprise events
- Boltzmann brain – Philosophical thought experiment
- Bonferroni correction – Statistical technique used to correct for multiple comparisons
- Coincidence – Concurrence of events with no connection
- Infinite monkey theorem – Counterintuitive result in probability
- Junkyard tornado – Fallacious argument against abiogenesis of chance
- Law of small numbers
- Library of Babel – Short story by Jorge Luis Borges
- Littlewood's law – Statistical law
- Look-elsewhere effect – Statistical analysis phenomenon
- Miracle – Event not explicable by natural or scientific laws
- Murphy's Law – Adage that anything that can go wrong will go wrong
- Psychic phenomena – Person claiming extrasensory perception abilities
- Totalitarian principle – Quantum mechanics principle stating: "Everything not forbidden is compulsory"

Notes

1. Everitt 2002

2. Beitman, Bernard D., (15 Apr 2018), Intrigued by the Low Probability of Synchronicities? Coincidence theorists and statisticians dispute the meaning of rare events. (https://www.psychologytoday.com/us/blog/connecting-coincidence/201804/intrigued-the-low-probability-synchronicities) at PsychologyToday (https://www.psychologytoday.com)
3. Kida, Thomas E. (Thomas Edward) (2006). *Don't believe everything you think : the 6 basic mistakes we make in thinking*. Amherst, N.Y.: Prometheus Books. p. 97. ISBN 1615920056. OCLC 1019454221 (https://search.worldcat.org/oclc/1019454221).
4. Hand, David J. (February 1, 2014). "Math Explains Likely Long Shots, Miracles and Winning the Lottery [Excerpt]" (https://www.scientificamerican.com/article/math-explains-likely-long-shots-miracles-and-winning-the-lottery/). *Scientific American*.
5. "Institute of Mathematical Statistics | Hand writing: The Improbability Principle" (https://imstat.org/2015/02/16/hand-writing-the-improbability-principle/).
6. Mirsky, Steve (May 1, 2014). "Statistician David J. Hand Shows How the Seemingly Improbable Becomes a Sure Thing" (https://www.scientificamerican.com/article/statistician-david-j-hand-shows-how-the-seemingly-improbable-becomes-a-sure-thing/). *Scientific American*.
7. here other law of "Improbability principle" also acts - the "law of probability lever", which is (according to David Hand) a kind of butterfly effect: we have a value "close" to 1 raised to large number what gives "surprisingly" low value or even close to zero if this number is larger, this shows some philosophical implications, questions the theoretical models but it does not render them useless - evaluation and testing of theoretical hypothesis (even when probability of it correctness is close to 1) can be its falsifiability - feature widely accepted as important for the scientific inquiry which is not meant to lead to dogmatic or absolute knowledge, see: statistical proof.
8. Graphing calculator (https://www.desmos.com/calculator/r3nmu0qxli) at Desmos (graphing)
9. Proof in: Elemér Elad Rosinger, (2016), "Quanta, Physicists, and Probabilities ... ?" (https://hal.archives-ouvertes.fr/hal-01386163/document) page 28
10. Reliability of Systems in Concise Reliability for Engineers (https://books.google.com/books?id=aMGQDwAAQBAJ&pg=PA33), Jaroslav Menčík, 2016
11. 1980, Austin Society to Oppose Pseudoscience (ASTOP) distributed by ICSA (former American Family Foundation) "*Pseudoscience Fact Sheets, ASTOP: Psychic Detectives*" (http://www.icsahome.com/elibrary/studyguides/education/astoppsychicdetectives)
12. Daniel Freeman, Jason Freeman, 2009, London, *"Know Your Mind: Everyday Emotional and Psychological Problems and How to Overcome Them"* p. 41
13. Mikal Aasved, 2002, Illinois, *The Psychodynamics and Psychology of Gambling: The Gambler's Mind* vol. I, p. 129

References

- Weisstein, Eric W. "Law of truly large numbers" (https://mathworld.wolfram.com/LawofTrulyLargeNumbers.html). *MathWorld*.
- Diaconis, P.; Mosteller, F. (1989). "Methods of Studying Coincidences" (https://web.archive.org/web/20100712091914/http://stat.stanford.edu/~cgates/PERSI/papers/mosteller89.pdf) (PDF). *Journal of the American Statistical Association*. **84** (408): 853–61. doi:10.2307/2290058 (https://doi.org/10.2307%2F2290058). JSTOR 2290058 (https://www.jstor.org/stable/2290058). MR 1134485 (https://mathscinet.ams.org/mathscinet-getitem?mr=1134485). Archived from the original (http://stat.stanford.edu/~cgates/PERSI/papers/mosteller89.pdf) (PDF) on 2010-07-12. Retrieved 2009-04-28.
- Everitt, B.S. (2002). *Cambridge Dictionary of Statistics* (2nd ed.). ISBN 978-0521810999.

- David J. Hand, (2014), *The Improbability Principle: Why Coincidences, Miracles, and Rare Events Happen Every Day* (https://books.google.com/books?id=raZRAQAAQBAJ)

External links

- Math Explains Likely Long Shots, Miracles and Winning the Lottery (Excerpt) (http://www.scientificamerican.com/article/math-explains-likely-long-shots-miracles-and-winning-the-lottery/) in Scientific American by David Hand 2014
- skepdic.com on the *Law of Truly Large Numbers* (http://skepdic.com/lawofnumbers.html)
- on the *Law of Truly Large Numbers* (http://www.quackwatch.org/04ConsumerEducation/coincidence.html)
- The On-Line Encyclopedia of Integer Sequences (https://oeis.org/A219330) – related integer sequence